5 Operations on Rational Numbers

29LNLF

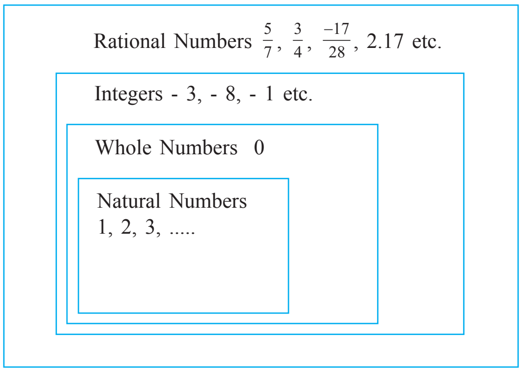
Let's learn.

Rational Numbers

In previous standards, we have learnt that the counting numbers 1, 2, 3, 4, ... are called natural numbers. We know that natural numbers, zero, and the opposite numbers of natural numbers together form the group of integers. We are also familiar with fractions like $\frac{7}{11}, \frac{2}{5}, \frac{1}{7}$. Is there then, a group that includes both integers and fractions? Let us see.

$4 = \frac{12}{3}$, $7 = \frac{7}{1}$, $-3 = \frac{-3}{1}$, $0 = \frac{0}{2}$ Thus, we also know that all integers can be written in the form $\frac{m}{n}$. If m is any integer and n is any non-zero integer, then the number $\frac{m}{n}$ is called a rational number.

This group of rational numbers includes all types of numbers mentioned before.



Complete the table given below.

	-3	$\frac{3}{5}$	-17	$-\frac{5}{11}$	5
Natural Number	×				✓
Integers	✓				
Rational Number	✓				

Operations on Rational Numbers

Rational numbers are written like fractions using a numerator and a denominator. That is why, operations on rational numbers are carried out as on fractions.

(1) $\frac{5}{7} + \frac{9}{11} = \frac{55+63}{77} = \frac{118}{77}$

(2) $\frac{1}{7} - \frac{3}{4} = \frac{4-21}{28} = \frac{-17}{28}$

(3) $2\frac{1}{7} + 3\frac{8}{14} = \frac{15}{7} + \frac{50}{14}$

$= \frac{30}{14} + \frac{50}{14}$

$= \frac{80}{14} = \frac{40}{7}$

(4) $\frac{9}{13} \times \frac{4}{7} = \frac{9 \times 4}{13 \times 7} = \frac{36}{91}$

(5) $\frac{3}{5} \times \frac{(-4)}{5} = \frac{3 \times (-4)}{5 \times 5} = \frac{-12}{25}$

(6) $\frac{9}{13} \times \frac{26}{3} = \frac{3 \times 2}{1} = \frac{6}{1}$

Let's recall.

To divide one number by another is to multiply the first by the multiplicative inverse of the other.

We have seen that $\frac{5}{6}$ and $\frac{6}{5}$, $\frac{2}{11}$ and $\frac{11}{2}$ are pairs of multiplicative inverses.

Similarly, $\left(\frac{-5}{4}\right) \times \left(\frac{-4}{5}\right) = 1$; $\left(\frac{-7}{2}\right) \times \left(\frac{-2}{7}\right) = 1$ Thus $\left(\frac{-5}{4}\right)$ and $\left(\frac{-4}{5}\right)$ as also $\left(\frac{-7}{2}\right)$ and $\left(\frac{-2}{7}\right)$ are pairs of multiplicative inverses. Similarly, $\frac{-5}{4}$ and $\frac{-4}{5}$ or $\frac{-7}{2}$ and $\frac{-2}{7}$ are pairs of multiplicative inverses. That is, $\frac{-5}{4}$ and $\frac{-4}{5}$ are each other's multiplicative inverses and so are $\frac{-7}{2}$ and $\frac{-2}{7}$.

Take care!

Example The product of $\frac{-11}{9}$ and $\frac{9}{11}$ is -1. Therefore, $\frac{-11}{9}$, $\frac{9}{11}$ is not a pair of multiplicative inverses.

Let's discuss.

Let us look at the characteristics of various groups of numbers. Discuss them in class to help you complete the following table. Consider the groups of natural numbers, integers and rational numbers. In front of each group, write the inference you make after carrying out the operations of addition, subtraction, multiplication and division, using a (✓) or a (×). Remember that you cannot divide by zero.

- If natural numbers are added, their sum is always a natural number. So, we put a tick (✓) under 'addition' in front of the group of natural numbers.
- However, if we subtract one natural number from another, the answer is not always a natural number. There are numerous examples like 7–10 = –3. So, under subtraction, we put a (×). If there is a cross in the table, explain why. To explain the reason for a (×), giving only one of the numerous examples is sufficient.

Group of numbers	Addition	Subtraction	Multiplication	Division
Natural numbers	✓	× (7 – 10 = – 3)	✓	× $(3 \div 5 = \frac{3}{5})$
Integers				
Rational numbers				

Now I know!

- The group of natural numbers is closed for the addition and multiplication but not for the subtraction and division. In other words, the difference of any two natural numbers or the quotient obtained on dividing one natural number by another will not always be natural number.
- The group of integers is closed for addition, subtraction and multiplication but not for division.
- The group of rational numbers is closed for all operations – addition, subtraction, multiplication and division. However, we cannot divide by zero.

Practice Set 22

1. Carry out the following additions of rational numbers.

   (i) $\frac{5}{36} + \frac{6}{42}$ (ii) $1\frac{2}{3} + 2\frac{4}{5}$ (iii) $\frac{11}{17} + \frac{13}{19}$ (iv) $2\frac{3}{11} + 1\frac{3}{77}$

2. Carry out the following subtractions involving rational numbers.

   (i) $\frac{7}{11} - \frac{3}{7}$ (ii) $\frac{13}{36} - \frac{2}{40}$ (iii) $1\frac{2}{3} - 3\frac{5}{6}$ (iv) $4\frac{1}{2} - 3\frac{1}{3}$

3. Multiply the following rational numbers.

   (i) $\frac{3}{11} \times \frac{2}{5}$ (ii) $\frac{12}{5} \times \frac{4}{15}$ (iii) $\frac{(-8)}{9} \times \frac{3}{4}$ (iv) $\frac{0}{6} \times \frac{3}{4}$

4. Write the multiplicative inverse.

   (i) $\frac{2}{5}$ (ii) $\frac{-3}{8}$ (iii) $\frac{-17}{39}$ (iv) 7 (v) $-7\frac{1}{3}$

5. Carry out the divisions of rational numbers.

   (i) $\frac{40}{12} \div \frac{10}{4}$ (ii) $\frac{-10}{11} \div \frac{-11}{10}$ (iii) $\frac{-7}{8} \div \frac{-3}{6}$ (iv) $\frac{2}{3} \div (-4)$

   (v) $2\frac{1}{5} \div 5\frac{3}{6}$ (vi) $\frac{-5}{13} \div \frac{7}{26}$ (vii) $\frac{9}{11} \div (-8)$ (viii) $5 \div \frac{2}{5}$

Let's learn. **Numbers in between Rational Numbers**

- Write all the natural numbers between 2 and 9.
- Write all the integers between −4 and 5.
- Which rational numbers are there between $\frac{1}{2}$ and $\frac{3}{4}$?

Example Let us look for rational numbers between the two rational numbers $\frac{1}{2}$ and $\frac{4}{7}$.

To do that, let us convert these numbers into fractions with equal denominators.

$$\frac{1}{2} = \frac{1\times 7}{2\times 7} = \frac{7}{14}, \qquad \frac{4}{7} = \frac{4\times 2}{7\times 2} = \frac{8}{14}$$

7 and 8 are consecutive natural numbers. But, are $\frac{7}{14}$ and $\frac{8}{14}$ consecutive rational numbers? The denominator of any number can be increased. Then the numerator also increases the same number of times.

$$\frac{7}{14} = \frac{70}{140}, \frac{8}{14} = \frac{80}{140} \; . \; . \; . \; . \; . \text{ (Multiplying the numerator and denominator by 10)}$$

Now, $\frac{70}{140} < \frac{71}{140} \; ..... < \frac{79}{140} < \frac{80}{140}$ So, how many numbers do we find between $\frac{7}{14}$ and $\frac{8}{14}$?

Also, $\frac{7}{14} = \frac{700}{1400}$, $\frac{8}{14} = \frac{800}{1400}$ ..... (Multiplying the numerator and denominator by 100)

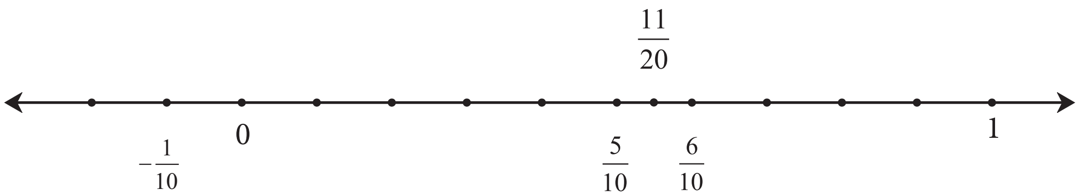
Hence, $\frac{700}{1400} < \frac{701}{1400} \; ...... < \frac{799}{1400} < \frac{800}{1400}$

Thus, when rational numbers are converted into equivalent fractions with increasingly bigger denominators, more and more rational numbers which lie between them can be expressed.

For example, let us find numbers between the rational numbers $\frac{1}{2}$ and $\frac{3}{5}$.

Let us first convert each of the numbers into their equivalent fractions.

For example, $\frac{1}{2} = \frac{5}{10}$, and $\frac{3}{5} = \frac{6}{10}$



On the number line, there are points showing the numbers $\frac{5}{10}$ and $\frac{6}{10}$. Let us find the mid-point of the line segment joining these two points and see what number it shows.

$$\frac{1}{2}\left(\frac{5}{10}+\frac{6}{10}\right) = \frac{11}{20}$$ This is the mid-point of the line segment.

Because $\frac{6}{10} - \frac{11}{20} = \frac{12-11}{20} = \frac{1}{20}$ and also $\frac{11}{20} - \frac{5}{10} = \frac{11-10}{20} = \frac{1}{20}$

Thus, $\frac{11}{20}$ is the number that lies exactly in the middle of $\frac{5}{10}$ and $\frac{6}{10}$. It means that, $\frac{11}{20}$ is a number that lies between $\frac{1}{2}$ and $\frac{3}{5}$. Similarly, we can find numbers that lie between $\frac{1}{2}$ and $\frac{11}{20}$ and between $\frac{11}{20}$ and $\frac{3}{5}$.

Now I know !

There are innumerable rational numbers between any two rational numbers.

Practice Set 23

⊙ Write three rational numbers that lie between the two given numbers.

(i) $\frac{2}{7}, \frac{6}{7}$ (ii) $\frac{4}{5}, \frac{2}{3}$ (iii) $-\frac{2}{3}, \frac{4}{5}$ (iv) $\frac{7}{9}, -\frac{5}{9}$

(v) $\frac{-3}{4}, \frac{+5}{4}$ (vi) $\frac{7}{8}, \frac{-5}{3}$ (vii) $\frac{5}{7}, \frac{11}{7}$ (viii) $0, \frac{-3}{4}$

Something more

If m is an integer, then $m + 1$ is the next bigger integer. There is no integer between m and $m + 1$. The integers that lie between any two non-consecutive integers can be counted. However, there are infinitely many rational numbers between any two rational numbers.

Let's recall.

We have learnt to multiply and divide decimal fractions.

$$\frac{35.1}{10} = 35.1 \times \frac{1}{10} = \frac{351}{10} \times \frac{1}{10} = \frac{351}{100} = 3.51$$

$$\frac{35.1}{100} = \frac{35.1}{1} \times \frac{1}{100} = \frac{351}{10} \times \frac{1}{100} = \left(\frac{351}{1000}\right) = 0.351$$

$$35.1 \times 10 = \frac{351}{10} \times 10 = 351.0$$

$$35.1 \times 1000 = \frac{351}{10} \times 1000 = \left(\frac{351000}{10}\right) = 35100.0$$

Thus we see that we can divide a decimal fraction by 100, by moving the decimal point two places to the left. To multiply by 1000, we move the point three places to the right.

The following rules are useful while multiplying and dividing.

No matter how many zeros we place at the end of the fractional part of a decimal fraction, and no matter how many zeros we place before the integral part of the number, it does not change the value of the given fraction.

$$1.35 = \frac{135}{100} \times \frac{100}{100} = \frac{13500}{10000} = 1.3500$$

$$1.35 = \frac{135}{100} \times \frac{1000}{1000} = \frac{135000}{100000} = 1.35000 \text{ etc.}$$

Also, see how we use : 1.35 = 001.35.

$$\frac{1.35}{100} = \frac{001.35}{100} = 0.0135$$

Let's learn. **Decimal Form of Rational Numbers**

Example Write the rational number $\frac{7}{4}$ in decimal form.

$$\begin{array}{r} 1.75 \\ 4\overline{)7.000} \\ -\ 4 \\ \hline 30 \\ -\ 28 \\ \hline 20 \\ -\ 20 \\ \hline 00 \end{array}$$

(1) 7 = 7.0 = 7.000 (Any number of zeros can be added after the fractional part.)

(2) 1 is the quotient and 3 the remainder after dividing 7 by 4. Now we place a decimal point after the integer 1. Writing the 0 from the dividend after the remainder 3, we divide 30 by 4. As the quotient we get now is fractional, we write 7 after the decimal point. Again we bring down the next 0 from the dividend and complete the division.

Example Write $2\frac{1}{5}$ in decimal form.

We shall find the decimal form of $2\frac{1}{5} = \frac{11}{5}$ in three different ways.

Find the decimal form of $\frac{1}{5}$.

(I)

$$\begin{array}{r} 0.2 \\ 5\overline{)1.0} \\ -\ 0 \\ \hline 10 \\ -\ 10 \\ \hline 00 \end{array}$$

$\frac{1}{5} = 0.2$

$\therefore\ 2\frac{1}{5} = 2.2$

(II)

$$\begin{array}{r} 2.2 \\ 5\overline{)11.000} \\ -\ 10 \\ \hline 010 \\ -\ 10 \\ \hline 00 \end{array}$$

$\frac{11}{5} = 2.2$

(III)

$$\frac{11}{5} = \frac{11 \times 2}{5 \times 2} = \frac{22}{10} = 2.2$$

Example Write the rational number $\frac{-5}{8}$ in decimal form.

The decimal form of $\frac{5}{8}$ obtained by division is 0.625. $\therefore\ \frac{-5}{8} = -0.625$

In all the above examples, we have obtained zero as the remainder. This type of decimal form of a rational number is called the terminating decimal form.

Example Let us see how the decimal form of some rational numbers is different.

(i) Write the number $\frac{5}{3}$ in decimal form.

```
   1.66
3)5.00
 - 3
  ----
   2 0
 - 1 8
  ----
    20
  - 18
  ----
     2
```

$\therefore \frac{5}{3} = 1.666.......$

$\therefore \frac{5}{3} = 1.\dot{6}$

(ii) Write the number $\frac{2}{11}$ in decimal form.

```
     0.18
11)2.00
  - 0
  -----
    20
  - 11
  -----
     90
   - 88
  -----
     20
```

$\therefore \frac{2}{11} = 0.1818.......$

$\therefore \frac{2}{11} = 0.\overline{18}$

(iii) Find the decimal form of $2\frac{1}{3}$. $2\frac{1}{3} = \frac{7}{3}$

```
   2.33
3)7.00
 - 6
  -----
   10
 -  9
  -----
    10
 -   9
  -----
    01
```

$2\frac{1}{3} = 2.33...$

$\therefore 2\frac{1}{3} = 2.\dot{3}$

(iv) Work out the decimal form of $\frac{5}{6}$.

```
  0.833
6)50
 - 48
  -----
   020
 -  18
  -----
    020
 -   18
  -----
     02
```

$\frac{5}{6} = 0.833...$

$\therefore \frac{5}{6} = 0.8\dot{3}$

In all the above examples, the division does not come to an end. **Here, a single digit or a group of digits occurs repeatedly on the right of the decimal point. This type of decimal form of a rational number is called the recurring decimal form.**

If in a decimal fraction, a single digit occurs repeatedly on the right of the decimal point, we put a point above it as shown here. $2\frac{1}{3} = 2.33... = 2.\dot{3}$, and if a group of digits occurs repeatedly, we show it with a horizontal line above the digits.

Thus, $\frac{2}{11} = 0.1818........= 0.\overline{18}$ and $\frac{5}{6} = 0.8\dot{3}$

Now I know!

Some rational numbers have a terminating decimal form, while some have a recurring decimal form.

Let's discuss.

- Without using division, can we tell from the denominator of a fraction, whether the decimal form of the fraction will be a terminating decimal? Find out.

Practice Set 24

⊙ Write the following rational numbers in decimal form.

(i) $\frac{13}{4}$ (ii) $\frac{-7}{8}$ (iii) $7\frac{3}{5}$ (iv) $\frac{5}{12}$ (v) $\frac{22}{7}$ (vi) $\frac{4}{3}$ (vii) $\frac{7}{9}$

Let's discuss.

An arrangement of numbers expressed using the signs of addition, subtraction, multiplication and division is called a mathematical expression.

Simplify the expression $72 \div 6 + 2 \times 2$ and find its value.

Hausa's method

$72 \div 6 + 2 \times 2$

$= 12 + 2 \times 2$

$= 12 + 4$

$= 16$

Mangru's Method

$72 \div 6 + 2 \times 2$

$= 12 + 2 \times 2$

$= 14 \times 2$

$= 28$

We got two different answers because the two children carried out the operations in different orders. To prevent this, some rules have been made which decide the order in which the operations are carried out. If these rules are followed, we will always get the same answer. When an operation is to be carried out first, it is shown using brackets in the expression.

Rules for Simplifying an Expression

(1) If more than one operation is to be carried out, then multiplication and division are carried out first, in the order in which they occur from left to right.

(2) After that, addition and subtraction are carried out in the order in which they occur from left to right.

(3) If there are more than one operations in the brackets, follow the above two rules while carrying out the operations.

On applying the above rules, we see that Hausa's method is the right one.

$\therefore\ 72 \div 6 + 2 \times 2 = 16$

Let's evaluate the expressions below.

Example $40 \times 10 \div 5 + 17$
$= 400 \div 5 + 17$
$= 80 + 17$
$= 97$

Example $80 \div (15 + 8 - 3) + 5$
$= 80 \div (23 - 3) + 5$
$= 80 \div 20 + 5$
$= 4 + 5$
$= 9$

Example $2 \times \{25 \times [(113 - 9) + (4 \div 2 \times 13)]\}$

$= 2 \times \{25 \times [104 + (4 \div 2 \times 13)]\}$

$= 2 \times \{25 \times [104 + (2 \times 13)]\}$

$= 2 \times \{25 \times [104 + 26]\}$

$= 2 \times \{25 \times 130\}$

$= 2 \times 3250$

$= 6500$

Example $\frac{3}{4} - \frac{5}{7} \times \frac{1}{3}$

$= \frac{3}{4} - \frac{5}{21}$ (multiplication first)

$= \frac{3\times21-5\times4}{84}$ (then, subtraction)

$= \frac{63-20}{84}$

$= \frac{43}{84}$

Remember :

Brackets may be used more than once to clearly specify the order of the operations. Different kinds of brackets such as round brackets ( ), square brackets [ ], curly brackets { }, may be used for this purpose.

When solving brackets, solve the innermost bracket first and follow it up by solving the brackets outside in turn.

Practice Set 25

- Simplify the following expressions.

1. $50 \times 5 \div 2 + 24$
2. $(13 \times 4) \div 2 - 26$
3. $140 \div [(-11) \times (-3) - (-42) \div 14 - 1)]$
4. $\{(220-140) + [10 \times 9 + (-2 \times 5)]\} - 100$
5. $\frac{3}{5} + \frac{3}{8} \div \frac{6}{4}$

Activity : Use the signs and numbers in the boxes and form an expression such that its value will be 112.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9	+ × ÷ –

Something more

The order of the signs when simplifying an expression

B →	O →	D →	M →	A →	S
( )	×	÷	×	+	–
First, the operations in the brackets	Of multiplication e.g. $\frac{1}{4}$ of 200 $= 200 \times \frac{1}{4}$	Division	Multiplication	Addition	Subtraction